

Gravitation as a Many Body Problem

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The idea of viewing gravitation as a many body phenomenon is put forward here. Physical arguments supporting this idea are briefly reviewed. The basic mathematical object of the new gravitational mechanics is a matrix of operators [1]. Striking similarity of the method of R-matrix (QISM) to the mathematical formulation of the new gravitational mechanics is pointed out. The s-wave difference Schrodinger equation describing a process of emission of radiation by a gravitating particle [1] is shown to be analogous to the Baxter equation of the QISM [2].

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I. INTRODUCTION

It is well known that the classical gravitating systems behave in the way foreign to statistical quantum mechanics. The negative specific heat of those systems and the phenomenon of gravitational collapse are different facets of the same reality. The difficulties arise which necessitate that the complete atomistic description of gravitation be sought after [1,4]. An effective spacetime description of phenomena would be than obtained in the thermodynamical large N limit [4,5]. The well known paradoxes of gravitational physics which only until recently stood apart are recognized now as intimately connected [1,4,5]. They are: *i)* an ultimate gravitational collapse which also leads to the breakdown of spacetime description; *ii)* a negative specific heat of gravitating systems and of black holes specifically [6,7]; *iii)* unitarity nonconservation on black holes [7,8]; *iv)* information loss on black holes, paradox of nonorthodox causality structure, “breakdown of physics” in singularities; *v)* nonrenormalizability of gravitation in the usual scheme of QFT. The well known fact of nonrenormalizability of gravitation is not surprising after all because gravitons are not the right microscopic degrees of freedom. This is here where the *non-collapse postulate* and the resulting statistical mechanical arguments [4,5] help to establish that the microscopic degrees of freedom are the Planckian mass *gravitational atoms* rather than gravitons. Gravitational quanta [4] and gravitons must then appear as collective excitations of a bound state of many gravitational atoms. That the

microscopic theory of gravitation must involve new and previously unanticipated degrees of freedom, which have nothing to do with strings or membranes [19,20], was first proposed several years ago [1,4,9]. The idea of introducing new kinematics, and to be specific, exactly, the generalization of the Heisenberg matrix mechanics to the matrix of operators for a gravitating particle was first described in [1] and at the USC Summer 1995 Institute lecture [9,18]. It should be clear that kinematics is fundamental whereas dynamics depends on a system: there are many Hamiltonians but only one fundamental relation $[p, q] = \frac{\hbar}{2\pi i}$. This is why my proposal for kinematics of the new gravitational mechanics [1] is universal while strings and membranes are only models [19,20].

II. THE FUNDAMENTAL IDEA OF THE NEW GRAVITATIONAL MECHANICS

I will briefly present different heuristic arguments which have led me to the proposal of the new gravitational mechanics [1]. In essence my argument that the fundamental object of gravitational mechanics is the matrix of operators followed from a careful examination of the GRT Kepler problem [1]. The geodesic motion is uniformized by a complex symplectic torus. Hence the idea of a noncommutative torus replacing a symplectic torus has emerged [9,1]. Two sets of matrices of operators $\mathbf{X}^\mu, \mathbf{P}_\mu$, which now describe a geodesic motion of a test particle satisfy a matrix Heisenberg relations [9,10].

There exists a subtle connection between the idea of a matrices of operators describing a gravitating particle and QISM. The second symplectic structure of the KdV equation is the same as the Virasoro algebra, the algebra of classical reparametrization invariance of a geodesic action principle. Quadratic noncommutative algebras related to the Yang-Baxter equation are central to the quantum inverse scattering method (QISM) [2]. QISM or R-matrix method deals directly with a matrix of operators as the fundamental object [2].

I have noticed long time ago that a classical geodesic motion of a test particle (massive or massless) in the gravitational field of a Schwarzschild black hole is related to the single soliton solution of the KdV equation. This is so because

$$u = r^{-1} = \frac{1}{2KM} \left(\frac{1}{3} + 4\mathcal{P}(\phi; \omega, \omega') \right), \quad (1)$$

where $K = \frac{G}{c^2}$ is the Einsteinian gravitational constant, and M is the mass of a black hole, is a solution to the geodesic equation. This motion is described by the doubly periodic Weierstrass \mathcal{P} -function, with two periods $2\omega, 2\omega'$ which determine a lattice on the complex ϕ plane. The classical geodesic motion in the Schwarzschild black hole gravitational field is uniformized by a complex torus. I have suggested that this complex torus be regarded as a symplectic torus. This has led me to the canonical Sommerfeld phase integral

$$I = \oint p_r dr \quad (2)$$

over a homotopically nontrivial contour on an elliptic curve uniformizing the actual physical motion (double covering of the complex r -plane). Similar construction can be easily carried through for the geodesic motion in the Kerr black hole gravitational field with the effect that

$$I = 4\pi i G \delta M_C^2 = i\delta \frac{A_{bh}}{4G} = i\delta S_{bh}, \quad (3)$$

where M_C is the Christodoulou irreducible mass¹ [11], A_{bh} is an area of a black hole horizon, and S_{bh} is a black hole entropy [6,7] ($\hbar = c = k = 1$ is assumed here). Semiclassical quantization conditions of Sommerfeld can be easily extended to the case of the complexified KAM torus (complexified integrable systems) [1,9,10]. The two angle variables on a complex uniformizing torus lead to the generalized Heisenberg matrix mechanics. It is essential here that the two periods are distinct in character, as one of them is complex (purely imaginary for bound orbits). I have followed the basic idea of transition to matrix mechanics as described by Dirac [3,1]. According to Dirac [3] the fundamental idea of matrix mechanics was to consider *two Bohr orbits instead of one*. Now, two distinct periods in the GRT Kepler problem suggested to me the idea of considering two sets of two integers related to two periods on the complex uniformizing torus, and hence a matrix of matrices. This is the essence of the idea of generalized Heisenberg matrix mechanics applied to gravitation: a matrix of operators \mathcal{O}_{mn} as the fundamental object. The problem of a particle moving in the gravitational field of two heavy centers (two extremally charged black holes described classically by the Majumdar-Papapetrou metric), which is the GRT Pauli problem, shows that one must now consider an object $\mathcal{O}_{\mathbf{mn}}$, where \mathbf{n} is a vector multi index; in the Pauli problem this vector index has only two components. The generalization to other systems is obvious [10]. The action principle for the GRT Kepler and Pauli problems is the generalization of the geodesic action principle for the mathematical object $\mathbf{X}^\mu_{\mathbf{mn}}$:

$$I = \frac{1}{2l^2} \int Tr(\mathbf{g}_{\mu\nu}(\mathbf{X}) \partial \mathbf{X}^\mu \partial \mathbf{X}^\nu) d\xi, \quad (4)$$

where l is the Planck length, $\mathbf{g}_{\mu\nu}$ is a metric tensor which is now a matrix of operators, and $\partial \mathbf{X} = \frac{d}{d\xi} \mathbf{X}$, with ξ a complex parameter. The trace operation $Tr(\mathbf{AB})$ is defined as a trace over two vector spaces on which a matrix of matrices (operators) acts. In this way one recovers in an explicit mathematical form an early heuristic suggestion of quantization of a black hole area originally due to Bekenstein [12–15]. It becomes clear now that the spacetime picture associated with

¹The Christodoulou irreducible mass is defined by the formula $M^2 = M_C^2 + \frac{J^2}{4G^2 M_C^2}$.

the surface of a black hole horizon divided into many Planckian area cells is quite misleading. Such a spacetime picture, sometimes called *the holographic principle* [12,14,15], is equally incorrect as a toy idea of the noncommutative configuration space would be at the time of the old quantum theory. It is the phase space, or symplectic geometry, which becomes noncommutative upon transition to quantum mechanics. Physical arguments based on the *non-collapse* postulate and on the s-wave difference Schrodinger equation are compatible. We have found a spectrum of collective excitations of a system of gravitational atoms [1,4,5].

III. THE NON-COLLAPSE POSTULATE AND INTEGRABLE MANY BODY SYSTEMS

The method of R-matrix or the quantum inverse scattering method (QISM) has proven quite powerful in the many body problem solving [2]. It is perhaps not too naive to expect that this method could be also applied to the phenomenon of the universal gravitation once it is demonstrated that a gravitational mass behaves as a many body system.

That gravitation is a many body problem follows from the black hole thermodynamics with the non-collapse hypothesis incorporated explicitly [1,4,5]. The Bekenstein entropy [6] of a black hole (and its negative specific heat) does not really imply that gravitation is actually a many body problem. Even though invoking the concept of entropy in the context of black holes implicitly means that some kind of atomism is assumed, the many body (atomic) nature of a gravitational mass was not uncovered until only quite recently [4,5]. The concept of gravitational quanta and gravitational atoms has appeared quite naturally once the non-collapse hypothesis was introduced. Previously, the information theoretic interpretation of entropy was adopted in accord with an apparent loss of information inside black holes [6]. Therefore, the concept of gravitational atoms and gravitational quanta did not appear earlier. On the other hand elementary planckian cells of black hole horizon area were almost always mentioned in the context of black hole thermodynamics [6,12,8,14,15]. Clearly, it is a long way from the concept of the black hole horizon area cells to the idea that the physical nature of a gravitational mass is best described as a many body problem. The arguments to this effect are of physical character and were first discovered several years ago [1,4,13]. This is to say, that our method is model independent and this conclusion was obtained long before the so-called D-brane soliton approach to certain unphysical black holes was presented in the literature [16].

The central character of the non-collapse hypothesis in the demonstration that gravitation is a many body phenomenon is difficult to overestimate. The *gedanken experiment* presented in the next section was a basis of our hypothesis that the total specific heat of a quantum black hole is positive [4,5]. This is clearly connected to the problem of unitarity conservation. With the help of this single postulate we were able to demonstrate that a gravitational mass-energy behaves as a number

of correlated harmonic oscillators [4,5]. This is in a complete agreement with the s-wave difference Schrodinger equation presented in [1]. I have called these correlated harmonic oscillators *gravitational quanta* because they are related to the gravitational mass.

Now, it became clear to me that the simplest properties of gravitational quanta (and gravitational masses) suggest that they are weakly coupled for a large total mass-energy E and that the total mass-energy of this system is a sum of energies of collective excitations

$$E = \sum_n \epsilon_n. \quad (5)$$

In particular, for a large natural number N a typical energy scale for collective excitations is

$$\epsilon_n \sim \frac{\mu}{\sqrt{N}}, \quad (6)$$

where μ is a Planck mass. The total mass-energy scales with N as follows:

$$E \sim \mu\sqrt{N}. \quad (7)$$

This is a general property of integrable quantum many body systems that a total energy is a sum over energies of collective excitations [2]. In my previous work on the collective excitations of a black hole I have proposed an s-wave difference Schrodinger wave equation which describes energy levels of a black hole [1]. I have taken a null shell model for the s-wave scalar particle mode in the geometric optics approximation, which is only adequate near a black hole horizon, and I have quantized the collective degree of freedom of such a shell. The energy spectrum came out a harmonic oscillator spectrum. Such a procedure is not quite unique, as the more detailed analysis has shown. On the other hand the validity of such a model is limited only to the region near a black hole horizon, and as such this model should be considered an approximation. One can conclude from this that membrane models are not to be considered valid unless qualitative agreement is reached with other independent arguments of physical nature as the one presented above. We should keep in mind that we move on a new ground here and we should seek the consistency of arguments. The formal analogy which comes to mind is based on two observations:

i) Gravitation is a many body phenomenon; the total mass-energy of a gravitating mass is a sum of energies of collective excitations.

ii) Quantum integrable models of many body systems are leading to the same property of the total energy, and in addition to that the method of separation of variables (SOV) [2] applied to such systems leads to the Schrodinger wave equation, called in this case the Baxter equation, which, in general, is a difference equation.

Our basic observation is that the s-wave Schrodinger equation describing collective excitations of a black hole [1]:

$$x(\psi(x + iL) + \psi(x - iL)) = (x + 2\epsilon)\psi(x), \quad (8)$$

where ϵ is an energy of a collective excitation and $L \sim \frac{1}{M}$ in Planck units, is of the same type as Baxter equations for collective excitations of integrable quantum many body systems [2]. The question which occurs naturally now is the following: What kind of mathematical structure describing a gravitating particle leads to the difference Schrodinger equation presented in [1] ? If we look closer into the derivation of the Baxter equations in the QISM approach of Sklyanin [2], we also notice that the basic mathematical structure is the matrix of operators constrained by some quadratic relations. Associativity of such quadratic algebras leads to the Yang-Baxter equations (YBE). Now, my idea of the second Heisenberg algebra related to the presence of the second period of Nature (the second angle variable on a complex torus) in the GRT Kepler problem [1] was to consider a matrix of operators as an observable in the new gravitational mechanics [1,10]. This observation has a profound meaning, and it cannot be just an accident, as it is based on physical arguments. One must follow the lead which this connection has opened up [10].

IV. THE GEDANKEN EXPERIMENT AND THE NON-COLLAPSE HYPOTHESIS

Consider the following situation. A large number of identical gravitating particles (mini black holes), which for simplicity will be assumed to be spinless, are contained in a large box. This large box can be considered to be filled up with a black body radiation at some temperature T . This cavity black body radiation is considered to be in thermodynamical equilibrium with a cavity walls which are also kept at a constant temperature T . The next step is to assume that a black body radiation is in statistical equilibrium with mini black holes. Unless small identical black holes are in statistical equilibrium among themselves due to collision processes the second law of thermodynamics is violated. The *non-collapse hypothesis* can be concisely formulated as the statement to the effect that in the processes of collisions between identical mini black holes inelastic processes do not happen (hence non-collapse), or that collisions are elastic with a high probability. This is quite similar to the situation considered once by Einstein [17]. Our gedanken experiment is concerned with identical gravitating particles in thermodynamical equilibrium, which is, on the other hand, impossible according to general relativity (GRT). In this situation GRT predicts a merger of two or more mini black holes in the process of a head-on collision. The phenomenon of gravitational collapse precludes the possibility of statistical equilibrium to be achieved in processes of mini black hole collisions. Hence, the hypothesis of statistical equilibrium in our gedanken experiment is a qualitatively new element introduced into the physical theory. Gravitational collapse in the head-on collisions of mini black holes does not happen. If it does happen, as GRT predicts, then the second law of thermodynamics is not valid in the situation described by our gedanken experiment.

The classic argument of Einstein [17] applied by him to a classical gas of atoms in statistical equilibrium with a black body radiation and among themselves leads to the Planck distribution formula for radiation once the Bohr formula $E_m - E_n = h\nu_{mn}$ (and the postulate of spontaneous and stimulated emission of radiation) was assumed for the processes of absorption and emission of radiation by atoms. Reversing this classic argument, when applied to our gedanken experiment, must necessarily lead to the quantized mass-energy levels of mini black holes [12,13,1,4,5]. The new postulate was made in derivation of this conclusion. This is the non-collapse postulate which logically plays the same role in our gedanken experiment as the postulate of spontaneous emission of radiation has played in the derivation of Planck formula [17]. In our case we postulate the Planck formula but we want to argue for the Bohr formula. We have thus reached the conclusion that a gravitating mass behaves statistically as if it consisted of a huge number of gravitational quanta. These quanta could be identified with collective excitation of a large number of Planckian mass scale gravitational atoms [4,5]. Statistical considerations are qualitative by nature and they do not lead to the definitive mathematical models for the dynamics of those gravitational atoms. The only important statement which follows from our work [4,5] is the definitive argument about the physical nature of gravitating particles. The nature of a gravitational mass is now understood qualitatively. Einsteinian gravitation is by its nature the many body phenomenon.

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